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ϵ'/ϵ AND HEAVY TOP*

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Abstract

The article reviews the new theoretical developments for the CP-parameter ϵ'/ϵ and its intimate connection with the mass of a heavy top quark.

1. INTRODUCTION

In this article we review recent theoretical estimates of the CP-parameter (ϵ'/ϵ), which include the modifications introduced by a heavy top quark. CP violating effects have so far been established only in the K -meson system. They consist

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of the leptonic asymmetry^[1]

$$\delta = \frac{\Gamma(K_L \rightarrow e^+ \nu \pi^-) - \Gamma(K_L \rightarrow e^- \bar{\nu} \pi^+)}{\Gamma(K_L \rightarrow e^+ \nu \pi^-) + \Gamma(K_L \rightarrow e^- \bar{\nu} \pi^+)} = (3.30 \pm 0.12) \times 10^{-3} \quad (1)$$

and the hadronic decays^{†[1]}

$$\eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} = (2.266 \pm 0.018) \times 10^{-3} e^{i(44.6 \pm 1.2)^\circ} \quad (2)$$

$$\eta_{00} = \frac{A(K_L \rightarrow \pi^0 \pi^0)}{A(K_S \rightarrow \pi^0 \pi^0)} = (2.245 \pm 0.036) \times 10^{-3} e^{i(54 \pm 5)^\circ} \quad (3)$$

Each of the above quantities is an unambiguous manifestation of CP violation. These ratios by themselves, however, could not establish whether the CP asymmetries originate in the mass matrix or the decay amplitudes or both. The reason is that a CP asymmetry in one decay amplitude can be transformed away by the freedom, which occurs in the definition of the $|K^0\rangle$ state. More explicitly, the freedom which occurs in the definition of a $|K^0\rangle$, i.e.

$$|\tilde{K}_0\rangle = e^{i\alpha} |K^0\rangle \quad (U(1) - \text{transformation}) \quad (4)$$

with α -real, permits the introduction of a phase convention, where one of the decay amplitudes

$$\begin{aligned} A(K^0 \rightarrow \pi\pi, I=0) \\ A(K^0 \rightarrow \pi\pi, I=2) \end{aligned} \quad (5)$$

is real. Thus, in order to establish CP violation in the decay amplitudes, it is necessary to show that

$$\text{phase } A(K^0 \rightarrow \pi\pi, I=2) \neq \text{phase } A(K^0 \rightarrow \pi\pi, I=0). \quad (6)$$

[†]There are new results on the phase difference which provide a new test of the CPT theorem.

$$\phi_{00} - \phi_{+-} = -0.3 \pm 2.4 \pm 1.2^\circ \quad (E 731)$$

$$0.2 \pm 2.6 \pm 1.2^\circ \quad (NA 31)$$

This is precisely the purpose of the measurements for η_{+-} and η_{00} , but since nature has chosen a very small or perhaps zero phase difference, the subject is presently under active investigation (see eqs. (11) and (12) below). The relative phase is measured by the parameter ϵ' , which is defined as^[2]

$$\begin{aligned}\epsilon' &= \frac{i}{\sqrt{2}} \left[\text{Im} \left(\frac{A_2}{A_0} \right) \right] e^{i\Delta} \\ &= \frac{i}{\sqrt{2}} \left[\frac{\text{Im} A_2}{A_2} - \frac{\text{Im} A_0}{A_0} \right] e^{i\Delta}\end{aligned}\quad (7)$$

where Δ is the phase introduced by the final-state-interaction of the two pions. Introducing the fact that $\arg \epsilon \approx \arg \epsilon'$ we arrive at

$$\epsilon'/\epsilon = \frac{1}{\sqrt{2}} \frac{1}{|\epsilon|} \left(\frac{\text{Im} A_2}{A_0} - \omega \frac{\text{Im} A_0}{A_0} \right) \quad (8)$$

with $\omega = |A_2/A_0| = 1/22$. The second term is suppressed by a factor ω ; thus for the case

$$\text{Im} A_0 = 0 (G_F \alpha_{strong}) \quad (9)$$

and

$$\text{Im} A_2 = 0 (G_F \alpha_{em}) \quad (10)$$

the two terms in eq. (8) are comparable. This is in fact the case we shall consider, as indicated by the diagrams in fig. 1^[3].

Because of the physical significance of ϵ' , a large experimental effort has been launched to measure it. The NA 31 experiment at CERN reported a positive effect^[4]

$$(\epsilon'/\epsilon) = (3.3 \pm 1.1) \times 10^{-3} \quad (11)$$

which is 3σ away from zero. The E 731 experiment at Fermilab has analyzed 20% of their data and does not confirm the above result. It reports^[5]

$$(\epsilon'/\epsilon) = (-0.4 \pm 1.4 (\text{stat.}) \pm 0.6 (\text{syst})) \times 10^{-3} \quad (12)$$

Thus, the final value for the ratio is still an open issue and should be clarified by further analysis of the data and refinements of the results. In this situation it is important to sharpen the theoretical predictions for the ratio.

What we need are theoretical estimates for ImA_0 and ImA_2 . Phases for the amplitudes are introduced by Feynman diagrams, like the penguins and others. To derive an effective Hamiltonian at low energies, $p^2 \approx m_h^2$, we follow the method of integrating out the heavy quarks^[6]. One computes at the high energy scale m_w all relevant diagrams, whose values will be used later on as initial conditions. Strong and electroweak corrections to the original operators generate new operators, which after higher order corrections reproduce themselves. Next, one integrates the renormalization group equations between thresholds dropping at each stage the heavy quarks. Thus one arrives, at the end, at an effective hamiltonian

$$H_{eff}(\mu) = -\frac{G}{\sqrt{2}} \sum_{\substack{i=1 \\ i \neq 4}}^8 \left\{ \xi_c C_i^c(\mu) + \xi_t C_i^t(\mu) \right\} Q_i \quad (13)$$

with

C_i^c, C_i^t : Renormalized coefficient functions,

Q_i : Basis of operators, and

$\xi_q = V_{qd} V_{qs}^*$ with V_{ij} elements of the KM matrix.

The KM matrix elements contain phases which produce imaginary parts. The unitarity of the KM matrix implies

$$Im\xi_c = -Im\xi_t \approx -\beta\gamma \sin \delta'$$

and gives

$$ImH_{eff}(\mu) = -\frac{G}{\sqrt{2}} Im\xi_c \sum_i C_i(\mu) Q_i \quad (14)$$

with $C_i(\mu) = C_i^c(\mu) - C_i^t(\mu)$. Substituting the effective Hamiltonian into the defini-

tion of the ϵ' , one obtains

$$\epsilon'/\epsilon = \frac{1}{\sqrt{2}} \frac{1}{|\epsilon|} \frac{1}{|A_0|} \frac{G}{\sqrt{2}} Im\xi_c \sum_{\substack{i=1 \\ i \neq 4}}^8 C_i(\mu) [\langle Q_i \rangle_2 - \omega \langle Q_i \rangle_0] . \quad (15)$$

The problem is now divided in two parts:

1. determination of the coefficient functions $C_i(\mu)$ at a low energy scale μ , and
2. estimates of the hadronic matrix elements $\langle \pi\pi | Q_i | K^0 \rangle$.

2. THE COEFFICIENT FUNCTIONS

The renormalization program follows standard methods^[6]. One starts at a high energy scale, $p = m_w$ and computes all quark diagrams, which contribute to the $\Delta S = 1$ hamiltonian. The relevant diagrams are presented in figure 1. Diagrams (1a) and (1b) are the lowest order weak interaction and the QCD corrected diagram, respectively. Diagram (1c) is the gluonic-penguin. The diagrams in fig. (1d) are of electroweak origin and became interesting because they increase^[7] with the top quark mass.^[8] They are referred to as the electroweak penguins. Finally, the diagrams in fig. (1e) are box diagrams with top quarks in the intermediate states. These diagrams and their gluonic corrections generate a set of twelve operators, which reproduce themselves when higher order QCD corrections are included.

A complete set of operators is the following:

$$\begin{aligned} Q_1 &= (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} && \text{Lowest order weak} \\ Q_2 &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A} && \text{and its gluonic corrections} \\ Q_3 &= (\bar{s}_\alpha d_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A} \\ Q_4 &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A} && \text{Gluon - penguin} \\ Q_5 &= (\bar{s}_\alpha d_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A} \end{aligned}$$

$$\begin{aligned}
Q_6 &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_7 &= \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V+A} \quad \text{Electroweak penguin} \\
Q_8 &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_9 &= (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} + (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} - \\
&\quad (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta d_\beta)_{V-A} - (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{c}_\beta c_\beta)_{V-A} \\
Q_{10} &= (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} - (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} - \\
&\quad (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta d_\beta)_{V-A} + (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{c}_\beta c_\beta)_{V-A} \\
Q_{11} &= (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{b}_\beta b_\beta)_{V-A} \quad \text{From box diagrams} \\
Q_{12} &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{b}_\beta b_\alpha)_{V-A}
\end{aligned} \tag{16}$$

The indices α and β refer to colours and a summation over repeated indices is assumed. The calculation of the diagrams at $p \approx m_w$ gives the initial conditions $C_i(m_w)$ for the subsequent integration of the renormalization group equations. During the past year several papers appeared which consider the effects of a heavy top quark and are summarized as follows.

1. A renormalization analysis of the coefficient C_1, \dots, C_6 was published by Schneider, Wu and myself^[9], ignoring the operators Q_7, \dots, Q_{12} . Our results showed that these coefficients are insensitive to the heavy top quark mass, but depend on the renormalization scale μ and Λ_{QCD} .
2. An independent article by Flynn and Randall^[7] included the operators Q_1, \dots, Q_{10} and found that the coefficients $C_7(p)$ and $C_8(p)$ are sensitive functions of m_t . In fact, as m_t increases, C_7 and C_8 change sign and enhance the significance of the electroweak terms. The above article was checked by two groups, inde-

pendent of each other,^[10,11] who pointed out an error in the initial conditions for the coefficients C_4, C_5 and C_7 . To be specific, the terms $\alpha_s D(\mu)$ should be replaced by $\alpha_s \otimes$ (the structure function for the gluon-penguin, i.e., $\alpha_s \cdot F(x)$ with $F(x)$ defined in eqs. (1) - (4) of ref. [11]). This modification changes the coefficients at low energies by $\sim 15\%$, to which the original authors concur. However, their original conclusion that the importance of the electroweak penguins operators are enhanced as m_t becomes larger than 100 GeV remains unaltered.

3. The work of Buchalla, Buras and Harlander^[10] also presents a detailed study of the parameter (ϵ'/ϵ) which decreases considerably for $m_t \gtrsim 100$ GeV. For very large values of $m_t > 200$ GeV the ratio could become negative.
4. All of the above articles omit the operators Q_{11} and Q_{12} because they expected their effects to be small. Recently Schneider, Wu and myself^[11] completed an analysis including Q_{11} and Q_{12} and found that they also modify the coefficients by $\sim 10\%$. In fact, their effects compensate somewhat the corrections described above. These effects are small, but still important because they determine the minimal value allowed for (ϵ'/ϵ) .

The calculation is also particular because Q_{12} is linearly dependent

$$\begin{aligned} Q_{12} &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{b}_\beta b_\alpha)_{V-A} \\ &= 2(Q_1 - Q_2) - Q_3 + Q_4 + Q_{10} + Q_{11} . \end{aligned} \tag{17}$$

and must be eliminated at all stages of the calculation. The effects of Q_{12} are still felt because it contributes to the initial conditions. The final results for the coefficients C_6, C_7 and C_8 are shown in figure 2. Our computation uses the rescaling method^[3] with independent operators at each energy scale. (See appendices A and B). We note that the variation of C_7 and C_8 with m_t is substantial. It is also worthwhile to note that the coefficient functions are sensitive functions of Λ_{QCD} .

3. HADRONIC MATRIX ELEMENTS

The effective low energy Lagrangian in (13) contains seven operators, whose matrix elements we should calculate for each process under consideration. The matrix elements $\langle Q_1 \rangle, \dots, \langle Q_6 \rangle$ were studied extensively for estimates of ϵ' and the $\Delta I = 1/2$ rule. Several methods were used:

1. Factorization^[12]
2. Chiral Lagrangian inspired by the $1/N$ expansion,^[13]
3. QCD sumrules,^[14] and
4. Lattice gauge theories.^[15]

In spite of the various names, these models have many points in common and it is hoped that their answers will eventually converge to the same values.

The interest here is on the new elements $\langle Q_7 \rangle_{0,2}$ and $\langle Q_8 \rangle_{0,2}$. In fact we can normalize them to $\langle Q_6 \rangle_0$ with the ratios being more reliable because several parameters drop out, i.e., the dependence on light quark masses. All the operators Q_5, \dots, Q_8 have a $(V - A)(V + A)$ Lorentz structure and are related after a Fierz rearrangement to scalar densities $\bar{q}_R^j q_L^i$. They are in turn given by the chiral Lagrangian^[13]

$$\bar{q}_R^j q_L^i = \frac{1}{4} f_\pi^2 r \left[U - \frac{1}{\Lambda^2} \partial^2 U \right]_{ij} \quad (18)$$

with

$$m_\pi^2 = \frac{1}{2} r (m_u + m_d) \quad \text{and} \quad (19)$$

$$m_K^2 = \frac{1}{2} r (m_d + m_s) \simeq \frac{1}{2} r m_s \quad . \quad (20)$$

A complete set of matrix elements is presented in the article by Buchalla, et al.,^[10] where the reader can find the relations

$$\langle Q_8 \rangle_0 / \langle Q_6 \rangle_0 = -\frac{1}{2} \frac{\Lambda_\chi^2}{m_K^2 - m_\pi^2} \quad (21)$$

$$\langle Q_8 \rangle_2 / \langle Q_8 \rangle_0 = -\frac{1}{2\sqrt{2}} \frac{\Lambda_x^2}{m_K^2 - m_\pi^2} \quad (22)$$

$$\langle Q_7 \rangle_0 / \langle Q_8 \rangle_0 = \langle Q_7 \rangle_2 / \langle Q_8 \rangle_2 = \frac{1}{3} \quad (23)$$

$$\langle Q_8 \rangle_0 = -4\sqrt{\frac{3}{2}} \left[\frac{m_K^2}{m_S(\mu) + m_d(\mu)} \right]^2 \frac{m_K^2 - m_\pi^2}{\Lambda_x^2} F_\pi \quad (24)$$

with $\Lambda_x = 1020$ MeV. The formulas (21) - (23) depend on quantities which are well known. If instead of the chiral Lagrangian we use the factorization method, as described in ref. [16], we obtain the same results for $\langle Q_8 \rangle_{0,2}$. Finally, since the QCD sumrules give a justification of the factorization results they will provide additional support for these relations.

In general, the matrix elements have a μ dependence, which is absent in the leading terms given above. To improve the situation we include for Q_1 and Q_2 the next to leading corrections. Values for the elements and the μ -dependence^[17] are given in Table 1.

μ in GeV	$\langle Q_1 \rangle_0$	$\langle Q_2 \rangle_0$	$\langle Q_1 \rangle_2$	$\langle Q_2 \rangle_2$
0.6	-0.022	-0.045	0.013	0.013
0.7	-0.024	-0.050	0.011	0.011
0.8	-0.026	-0.053	0.010	0.010

Table 1

Dependence of matrix element
on the renormalization scale.

4. NUMERICAL RESULTS

Now that we evaluated the coefficient functions and the matrix elements, there remains to combine them together and obtain the various contributions. In order to separate various dependences, we decided to normalize the results to $\langle Q_6 \rangle_0$

and present the various terms. In Tables 2a and b we show the contribution from $\Omega_{\eta+\eta'}$, which is independent of m_t , and each of the other terms at $\mu = 0.8$ GeV and for two values of $\Lambda_{QCD} = 0.1$ and 0.3 GeV, respectively.

m_t in GeV	Ω_8	$\Omega_{\eta+\eta'}$	Ω_{oct}	Ω_{27}	Ω_p	Ω_{EWP}	Ω_{tot}
50	1.00	-0.27	-0.10	0.01	-0.06	0.14	0.72
100	1.00	-0.27	-0.09	0.03	-0.05	-0.01	0.61
150	1.00	-0.27	-0.08	0.05	-0.05	-0.26	0.38
200	1.00	-0.27	-0.07	0.07	-0.05	-0.57	0.10
250	1.00	-0.27	-0.06	0.09	-0.04	-0.92	-0.22

Table 2a

Various terms contributing to (ϵ'/ϵ) for
 $\mu = 0.8$ GeV and $\Lambda_{QCD} = 0.1$ GeV

m_t in GeV	Ω_8	$\Omega_{\eta+\eta'}$	Ω_{oct}	Ω_{27}	Ω_p	Ω_{EWP}	Ω_{tot}
50	1.00	-0.27	-0.08	0.00	-0.04	0.04	0.66
100	1.00	-0.27	-0.07	0.01	-0.03	-0.08	0.56
150	1.00	-0.27	-0.06	0.02	-0.03	-0.31	0.35
200	1.00	-0.27	-0.05	0.03	-0.03	-0.75	0.08
250	1.00	-0.27	-0.05	0.04	-0.02	-0.91	-0.22

Table 2b

Terms contributing to ϵ'/ϵ for $\mu = 0.8$ GeV
and $\Lambda_{QCD} = 0.3$ GeV.

The notation here is as follows:

$$h_{oct} = \sum_{i=1}^2 C_i(\mu) \langle Q_i \rangle_0 \text{ and } h_{27} = -\frac{1}{\omega} \sum_{i=1}^2 C_i(\mu) \langle Q_i \rangle_2$$

$$h_i = C_i(\mu) \left(\langle Q_i \rangle_0 - \frac{1}{\omega} \langle Q_i \rangle_2 \right) \text{ for } i = 3, \dots, 8$$

and the Ω_i 's are normalized to h_8 as

$$\Omega_{oct} = \frac{h_{oct}}{h_8}, \dots, \Omega_p = \sum_{i=3}^5 h_i/h_8$$

and

$$\Omega_{EWP} = \sum_{i=7}^8 h_i/h_8.$$

The term $\Omega_{\eta+\eta'}$ is the long distance contribution from $\pi - \eta - \eta'$ mixing [18]. The terms which show a strong dependence on m_t are Ω_{27} and Ω_{EWP} . In fact, the latter starts with a small and positive value, then decreases and vanishes at $m_t \approx 80 - 100$ GeV and becomes negative for larger values of m_t . Thus, its role up to 100 GeV is to permit the negative contributions from $\Omega_{\eta+\eta'}$, Ω_{oct} and Ω_p to become apparent. For larger values of $m_t > 150$ GeV the Ω_{EWP} becomes large and negative reducing the sum of the terms further.

At this point it is important to pause and ask how reliable are the modifications and reductions introduced in Tables 2a and b. First, we remark that the choice of our presentation makes the results independent of the KM matrix elements, which will be introduced, at the end of this section as a multiplicative factor to Ω_{tot} . The bulk of the reduction in tables 2 comes from the terms $\Omega_{\eta+\eta'}$ and Ω_{EWP} . The long distance effects from $\pi - \eta - \eta'$ mixing were studied previously by several authors.^[18] We did not investigate it any further. The remaining terms including Ω_{EWP} , as was emphasized in Section 3, involve ratios of hadronic matrix elements and we do not expect large variations. However, there are non-factorizable terms and one should test the above expectations by lattice calculations. There remains to study the dependence of $\Sigma = h_8 \Omega_{tot}$ on the low energy renormalization scale μ . This is shown

in figure 3 for $\Lambda = 0.2$ GeV and $m_s = 0.175$ GeV. The large variation of Σ with m_t comes about from the new terms of electroweak origin. It is a substantial variation. The dependence on μ is very small. We also studied the dependence on m_s shown in figure 4. This dependence comes mostly from $\langle Q_6 \rangle_0$, which according to (24) has a $(1/m_s^2)$ dependence and brings in a larger uncertainty.

Finally, we must multiply Σ by $Im\xi_t = \beta\gamma \sin\delta'$ which depends on the angles and the phase δ' occurring in the KM matrix. The computation of ranges for $Im\xi_t$ is standard and relies on experimental constraints discussed by many groups [2,19] and in addition $|\epsilon_k|$ and the measured $B^0 - \bar{B}^0$ mixing [2,20]. Studies without (ϵ'/ϵ) as a constraint, find two solutions, one with $\delta > \frac{\pi}{2}$ and a second with $\delta < \frac{\pi}{2}$. Typical solutions occur in Ref. [10] fig. 4 and in an extensive study by Kim, et al. [21] in figs. (2) and (3d). The values for $Im\xi_t$ are correlated with other parameters and in particular m_t . What we shall use in this article are central values for $Im\xi_t$ and the extreme upper and lower bounds introduced in fig. 8 in order to show the larger range brought in by $Im\xi_t$. This is not the optimal method because we miss correlations of $Im\xi_t$, which will further restrict the range of (ϵ'/ϵ) . We plan to present a more detail study in the future^[11]. Other authors reported studies of the correlations^[10,12]. Figure 5 shows the results of Buchalla et al.^[10] for central values of the parameters: $B_k = 0.75, \beta = 0.005, \gamma = 0.05$. The three curves correspond to

1. Pure QCD case corresponding to $\alpha_{QED} = 0$.
2. The inclusion of $\Omega_{\eta+\eta'}$ and QED penguin contributions.
3. The full result after the Z^0 -penguin and box diagrams are included.

Figure (6) shows our calculation with the same inputs and the two results agree with each other. Figure (7) shows the changes introduced to figure (6) when m_s is reduced from 175 MeV down to 125 MeV. Finally, figure (8) shows the range for (ϵ'/ϵ) for the two extreme values of $Im\xi_t$. We note that the upper range of (ϵ'/ϵ) remains close to 2×10^{-3} but the lower values are significantly smaller. These

ranges depend on the value of m_t and the other parameters discussed for Σ . For $\Lambda_{QCD} = 0.2$ GeV we note that (ϵ'/ϵ) is positive for values of m_t less than 200 GeV.

5. CONCLUSIONS

The three groups which analyzed CP phenomena for a heavy top agree that (ϵ'/ϵ) could become very small for $m_t \gtrsim 77$ GeV. The general conclusion is that it can imitate the superweak theory only for the extreme case $m_t > 200$ GeV. The minimal value of (ϵ'/ϵ) is still under active consideration.

From our present understanding, I can draw the following conclusions:

- The significance of the electroweak terms is enhanced when the top quark is heavy [7,10,11].
- A complete renormalization analysis with the $1/N$ -estimates for matrix elements gives the range

$$2 \times 10^{-3} > \epsilon'/\epsilon > 1.0 \times 10^{-4} \text{ for } m_t < 200 \text{ GeV}$$

- There is still a large range for (ϵ'/ϵ) . It is important to study and improve the theoretical estimates.
- It is crucial to improve the experiments and bring them in agreement with each other, because for the ranges allowed for the parameter Λ_{QCD}, m_s and μ
 - A large value $\epsilon'/\epsilon \approx 1$ to 2×10^{-3} signifies an $m_t \approx 100$ GeV.
 - A smaller value $\epsilon'/\epsilon \sim 10^{-4}$ (consistent with zero) favors larger $m_t \gtrsim 200$ GeV [10,11,21] or the super weak theory.^[29]
 - Negative values for the ratio occur only at very heavy top quark masses; larger than 200 GeV.

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Figure Captions

- Figure 1: Diagrams contributing to the effective $\Delta S = 1$ Hamiltonian.
- Figure 2: The Wilson coefficient as functions of m_t and for the parameters shown in the figure.
- Figure 3: Dependence of Σ on m_t and μ .
- Figure 4: Dependence of Σ on m_t and m_s .
- Figure 5: The anatomy of ϵ'/ϵ with the curve “3” presenting the full result from figure (11) of ref. [10].
- Figure 6: The same as figure (5) from ref. [13].
- Figure 7: The same as in figure (5) for $m_s = 123$ MeV.
- Figure 8: Range for the ratio ϵ'/ϵ as a function of m_t for the extreme values of $Im \xi_t$.

A Determination of the Wilson coefficients

The wilson coefficients express QCD- and QED-corrections and are determined by solving the renormalization group equation (RGE)

$$[(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_q m_q \frac{\partial}{\partial m_q}) \delta_{ij} - \gamma_{ij}^T] C_j = 0 \quad (1)$$

stepwise from m_W to μ through the quark mass thresholds.

Here the β -function is $\beta(g) = -(33 - 2n_f) \frac{g^3}{48\pi^2}$ and γ_{ij} is the anomalous dimension matrix for each quark mass range.

The RGE solves the QCD-corrections to all orders in perturbation theory which is needed because the strong coupling constant isn't small.

A.1 Determination of the Real wilson coefficient C_i^c

The weak $\Delta S = 1$ -hamiltonian without QCD-corrections at the energy scale $Q = m_W$ assuming $m_t < m_W$ is

$$H_{wk}(m_W) = \frac{G_f}{\sqrt{2}} (\xi_u Q_u + \xi_c Q_c + \xi_t Q_t) \quad (2)$$

where $\xi_i = V_{is}^* V_{id}$ contains the Kobayashi-Maskawa-matrix elements and $Q_q = (\bar{s}_\alpha q_\alpha)_{V-A} (\bar{q}_\beta d_\beta)_{V-A}$. Using the unitarity of the Kobayashi-Maskawa-matrix $\xi_u = -(\xi_c + \xi_t)$ the weak hamiltonian becomes

$$H_{wk}(m_W) = -\frac{G_f}{\sqrt{2}} [\xi_c (Q_u - Q_c) + \xi_t (Q_u - Q_t)] \quad (3)$$

The initial condition $C_i^c(m_W)$ can be simply read off the first term of the weak hamiltonian H_{wk} by

$$Q_u - Q_c = \frac{1}{2} Q^+ + \frac{1}{2} Q^- \quad (4)$$

with

$$Q^\pm = (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \pm (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} - (u \rightarrow c) \quad (5)$$

The new operators Q^\pm are constructed in the way that they are multiplicatively renormalizable and that the penguin contribution cancel for them.

For $Q = m_c$ the charm-quark fields decouple, vanish from the operators and become

$$Q^\pm = (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \pm (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} = Q_2 \pm Q_1 \quad (6)$$

These operators are now not multiplicatively renormalizable, they get contribution from the penguin and mix with the penguin-operators. So the evaluation of the RGE gives the relative simple structure for the coefficient C_i^c :

$$\begin{aligned} C_i^c &= [X_{rq} (\frac{\alpha_s''(m_c^2)}{\alpha_s'''(\mu^2)})^{a_q'''} X_{qp}^{-1} D_p^{(+)}] (\frac{\alpha_s'(m_b^2)}{\alpha_s''(m_c^2)})^{a''(+)} (\frac{\alpha_s(m_W^2)}{\alpha_s'(m_b^2)})^{a'(+)} + \\ & [X_{rq} (\frac{\alpha_s''(m_c^2)}{\alpha_s'''(\mu^2)})^{a_q'''} X_{qp}^{-1} D_p^{(-)}] (\frac{\alpha_s'(m_b^2)}{\alpha_s''(m_c^2)})^{a''(-)} (\frac{\alpha_s(m_W^2)}{\alpha_s'(m_b^2)})^{a'(-)} \end{aligned} \quad (7)$$

where $D_p^\pm = (\pm \frac{1}{2}, \frac{1}{2}, 0, 0)$ is the initial condition read off the weak hamiltonian at $Q = m_c$, a_q''' contains the eigenvalues and the matrix X_{rq} the eigenvectors of the anomalous dimension matrix for 3 flavours.

A.2 Determination of the imaginary wilson coefficient C_i

Now we are interested in the imaginary wilson coefficients of the effective $\Delta S = 1$ -hamiltonian

$$\begin{aligned} \text{Im}H_{eff} &= -\frac{G_f}{\sqrt{2}} \sum_i (C_i^e \text{Im}(\xi_e) + C_i^t \text{Im}(\xi_t)) Q_i \\ &= -\frac{G_f}{\sqrt{2}} \text{Im}(\xi_t) \sum_i C_i Q_i \end{aligned} \quad (8)$$

where $\text{Im}(\xi_t) = -\text{Im}(\xi_e)$ is used and C_i is defined as the coefficient of the imaginary part and the relation between the coefficients holds $C_i = C_i^t - C_i^e$.

The initial condition for $C_i^t(m_W)$ arises from $Q_u - Q_t$ and the development of Q_t for $m_t > m_W$ to the gluon-, γ -, Z-penguin and box-diagram. These diagrams produce the operator basis $Q_1 \dots Q_{11}$ mentioned in the text. The box-diagrams with two top-quarks in the intermediate state produce the new operators $(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{b}_\beta b_\beta)_{V-A}$ and $(\bar{s}_\alpha d_\beta)_{V-A}(\bar{b}_\beta b_\alpha)_{V-A}$, where the last one can be expressed by

$$(\bar{s}_\alpha d_\beta)_{V-A}(\bar{b}_\beta b_\alpha)_{V-A} = 2Q_1 - 2Q_2 - Q_3 + Q_4 + Q_{10} + Q_{11} \quad (9)$$

which is considered in the initial condition and the anomalous dimension matrix. Subtracting C_i^e from section 1.1., the initial condition for the imaginary part at $Q = m_W$ is given by:

$$C_1(m_W) = \frac{\alpha}{2\pi} \left[\frac{1}{2 \sin^2 \theta_W} A(x) + \frac{10}{\sin^2 \theta_W} B(x) + \frac{4 \sin^2 \theta_W - 4}{\sin^2 \theta_W} C(x) + D(x) \right] \quad (10)$$

$$C_2(m_W) = 1 - \frac{\alpha}{4\pi \sin^2 \theta_W} A(x) \quad (11)$$

$$\begin{aligned} C_3(m_W) &= \frac{\alpha}{2\pi} \left[-\frac{1}{4 \sin^2 \theta_W} A(x) + \frac{-1}{\sin^2 \theta_W} B(x) + \frac{-\frac{2}{3} \sin^2 \theta_W + 1}{\sin^2 \theta_W} C(x) \right. \\ &\quad \left. - \frac{1}{6} D(x) \right] - \frac{\alpha_s(m_W^2)}{36\pi} G(x) \end{aligned} \quad (12)$$

$$C_4(m_W) = 3 \frac{\alpha_s(m_W^2)}{36\pi} G(x) + \frac{\alpha}{8\pi \sin^2 \theta_W} A(x) \quad (13)$$

$$C_5(m_W) = -\frac{\alpha_s(m_W^2)}{36\pi} G(x) \quad (14)$$

$$C_6(m_W) = 3 \frac{\alpha_s(m_W^2)}{36\pi} G(x) \quad (15)$$

$$C_7(m_W) = \frac{\alpha}{2\pi} \left[\frac{4}{3} C(x) + \frac{1}{3} D(x) \right] \quad (16)$$

$$C_8(m_W) = 0 \quad (17)$$

$$C_9(m_W) = -\frac{1}{2} + \frac{\alpha}{2\pi} \left[-\frac{5}{2 \sin^2 \theta_W} B(x) + \frac{1 - \sin^2 \theta_W}{\sin^2 \theta_W} C(x) - \frac{1}{4} D(x) \right] \quad (18)$$

$$\begin{aligned} C_{10}(m_W) &= -\frac{1}{2} - \frac{\alpha}{2\pi} \left[-\frac{1}{4 \sin^2 \theta_W} A(x) - \frac{5}{2 \sin^2 \theta_W} B(x) + \right. \\ &\quad \left. \frac{1 - \sin^2 \theta_W}{\sin^2 \theta_W} C(x) - \frac{1}{4} D(x) \right] \end{aligned} \quad (19)$$

$$C_{11}(m_W) = \frac{\alpha}{4\pi \sin^2 \theta_W} A(x) \quad (20)$$

with [4,5,6,7]

$$A(x) = -x \left(\frac{1}{4} + \frac{9}{4} \frac{1}{1-x} - \frac{3}{2} \frac{1}{(1-x)^2} \right) - \frac{3}{2} \left(\frac{x}{x-1} \right)^3 \ln x \quad (21)$$

$$B(x) = \frac{1}{4} \left(\frac{x}{1-x} + \frac{x}{(1-x)^2} \ln x \right) \quad (22)$$

$$C(x) = \frac{x}{8} \left(\frac{6-x}{1-x} + \frac{3x+2}{(1-x)^2} \ln x \right) \quad (23)$$

$$D(x) = \frac{1}{36(x-1)^3} (-19x^3 + 25x^2) + \quad (24)$$

$$\frac{1}{36(x-1)^4} (-6x^4 + 60x^3 - 108x^2 + 64x - 16) \ln x \quad (25)$$

$$G(x) = \frac{x}{(1-x)^3} \left(-\frac{9}{4} + \frac{11}{8}x + \frac{1}{8}x^2 \right) + \frac{1}{(1-x)^4} \left(1 - 4x + \frac{9}{4}x^2 \right) \ln x \quad (26)$$

When we evaluate the procedure, we shift the operator

$$Q_7 \dots Q_{10} \Rightarrow \frac{\alpha}{\alpha_s(m_W^2)} Q_7 \dots \frac{\alpha}{\alpha_s(m_W^2)} Q_{10} \quad (27)$$

how this was first done by ref. [2]. Then the initial conditions $C_7 \dots C_{10}$ have to be multiplied by $\alpha_s(m_W^2)/\alpha$.

The coefficient C_i is calculated by

$$C_r(\mu) = \sum_{k,n} \left[\sum_{p,q} X_{r,q} \left(\frac{\alpha_s(m_c^2)}{\alpha_s(\mu^2)} \right)^{a_q'''} X_{qp}^{-1} D_p^n \right] \left[\sum_{l,m} W_{nm} \left(\frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)} \right)^{a_n''} W_{ml}^{-1} E_l^h \right] \left[\sum_{i,j} V_{hj} \left(\frac{\alpha_s(m_W^2)}{\alpha_s(m_b^2)} \right)^{a_j'} V_{ji}^{-1} C_i(m_W) \right] \quad (28)$$

The matrix E_l^h and D_p^n contain the linear relations between the operators at each energy scale. So, in the development of the RGE from an energy scale m_W to μ , one reaches $Q = m_b$ where the b-quark field vanishes from theory and the operators. The consequence is that Q_{11} vanishes from theory and Q_{10} is related to $Q_{10} = -2Q_1 + 2Q_2 + Q_3 - Q_4$. The matrix E_l^h gets

$$-E_1^{10} = E_2^{10} = 2 \frac{\alpha}{\alpha_s'(m_b^2)} \quad (29)$$

$$E_3^{10} = -E_4^{10} = \frac{\alpha}{\alpha_s'(m_b^2)} \quad (30)$$

and otherwise ($l, k = 1, \dots, 9$)

$$E_l^h = \delta_{lh} \quad (31)$$

At an energy scale $Q = m_c$ the c-quark field vanishes from theory and from the operators. The matrix D_p^n becomes in accordance to $Q_9 = Q_1 + Q_2$ and $Q_4 = -Q_1 + Q_2 + Q_3$

$$D_2^9 = D_1^9 = \frac{\alpha}{\alpha_s''(m_b^2)} \quad (32)$$

$$-D_1^4 = D_2^4 = D_3^4 = 1 \quad (33)$$

and otherwise ($p, n = 1, \dots, 8; p, n \neq 4$)

$$D_p^n = \delta_{pn} \quad (34)$$

B Anomalous dimension matrices and some results

For the determination of the anomalous dimension matrices, we follow the approach of Bijnsens and Wise who present the matrices in a shifted operator basis. Our operator basis is now

$$Q_1 \dots Q_6, \frac{\alpha}{\alpha_s} Q_7 \dots \frac{\alpha}{\alpha_s} Q_{10}, Q_{11}$$

The anomalous dimension matrix for 5 flavours is

$$\gamma''' = \frac{g'''^2}{8\pi^2} \begin{pmatrix} -1 & 3 & 0 & 0 & 0 & 0 & \frac{8}{9} & 0 & -\frac{2}{3} & \frac{2}{3} & 0 \\ 3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & \frac{8}{27} & 0 & -\frac{2}{9} & \frac{2}{9} & 0 \\ 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & \frac{4}{27} & 0 & \frac{8}{9} & -\frac{8}{9} & 0 \\ 0 & 0 & \frac{22}{9} & \frac{2}{3} & -\frac{5}{9} & \frac{5}{3} & -\frac{20}{27} & 0 & \frac{14}{9} & \frac{4}{9} & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & \frac{16}{9} & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{19}{3} & \frac{4}{27} & \frac{4}{3} & -\frac{1}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{20}{3} & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{47}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{17}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{35}{3} & 0 \\ 6 & -6 & -3 & 3 & 0 & 0 & -\frac{4}{9} & 0 & \frac{1}{3} & -\frac{1}{3} + 3\frac{\alpha_s}{\alpha} & 2 \end{pmatrix} \quad (35)$$

For 4 flavours the anomalous dimension matrix is similar to that in [2] with an additional $4/3$ in γ_{57} and γ_{68} . For 3 flavours the matrix is identical to the matrix presented in [2].

From our calculation we present a characteristic result.

	$\Lambda_4 = 0.2 GeV; \mu = 0.8 GeV$						
m_t	C_1	C_2	C_3	C_5	C_6	C_7/α	C_8/α
50	0.043	-0.044	-0.018	0.010	-0.080	-0.068	-0.002
100	0.041	-0.044	-0.017	0.010	-0.090	-0.047	0.036
150	0.037	-0.044	-0.016	0.010	-0.097	0.008	0.113
200	0.033	-0.042	-0.014	0.010	-0.103	0.084	0.218
250	0.028	-0.041	-0.012	0.010	-0.110	0.180	0.348

Table 1: Imaginary Wilson Coefficients for $\Lambda_4 = 0.2$ GeV and $\mu = 0.8$ GeV.

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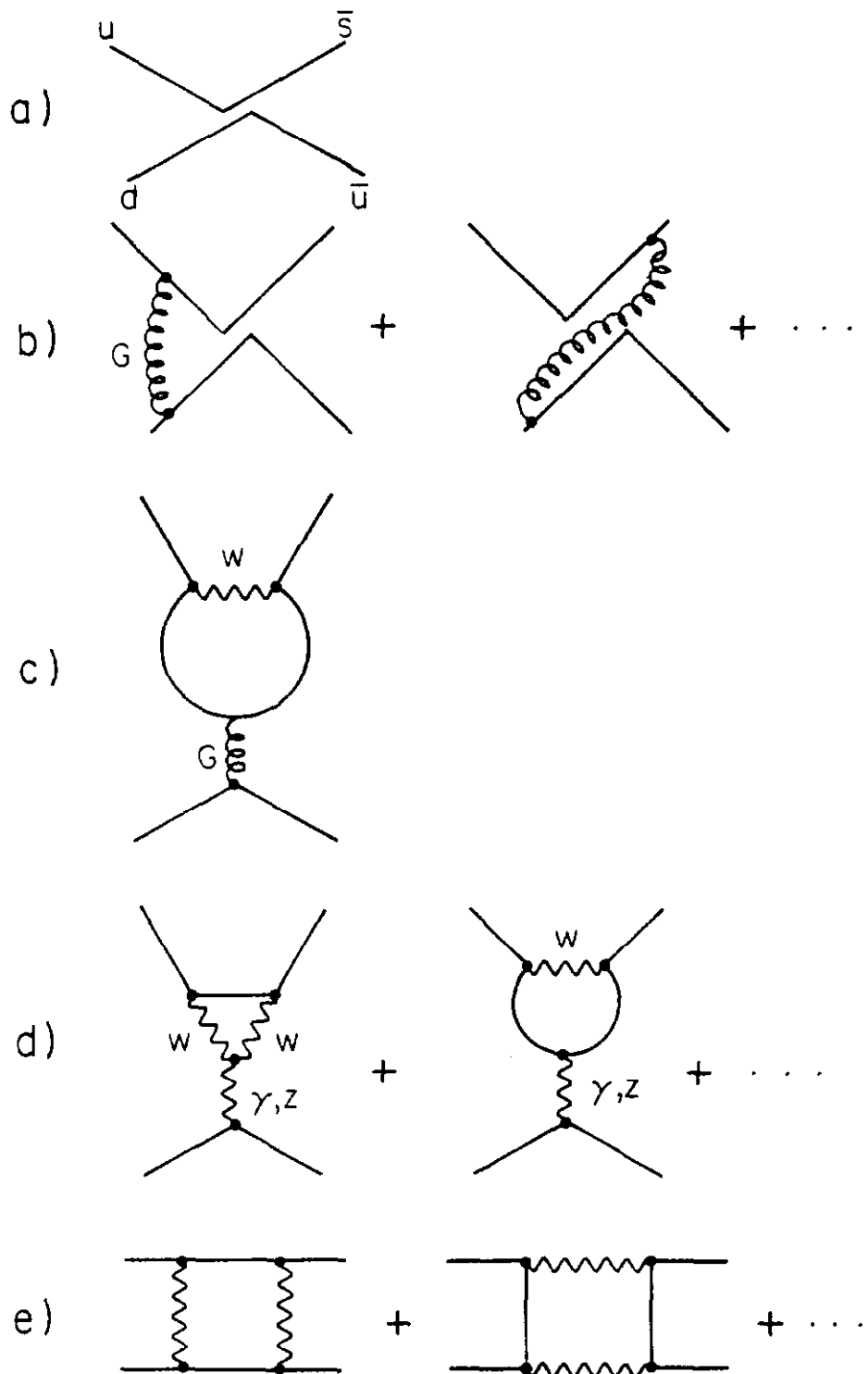


Fig. 1

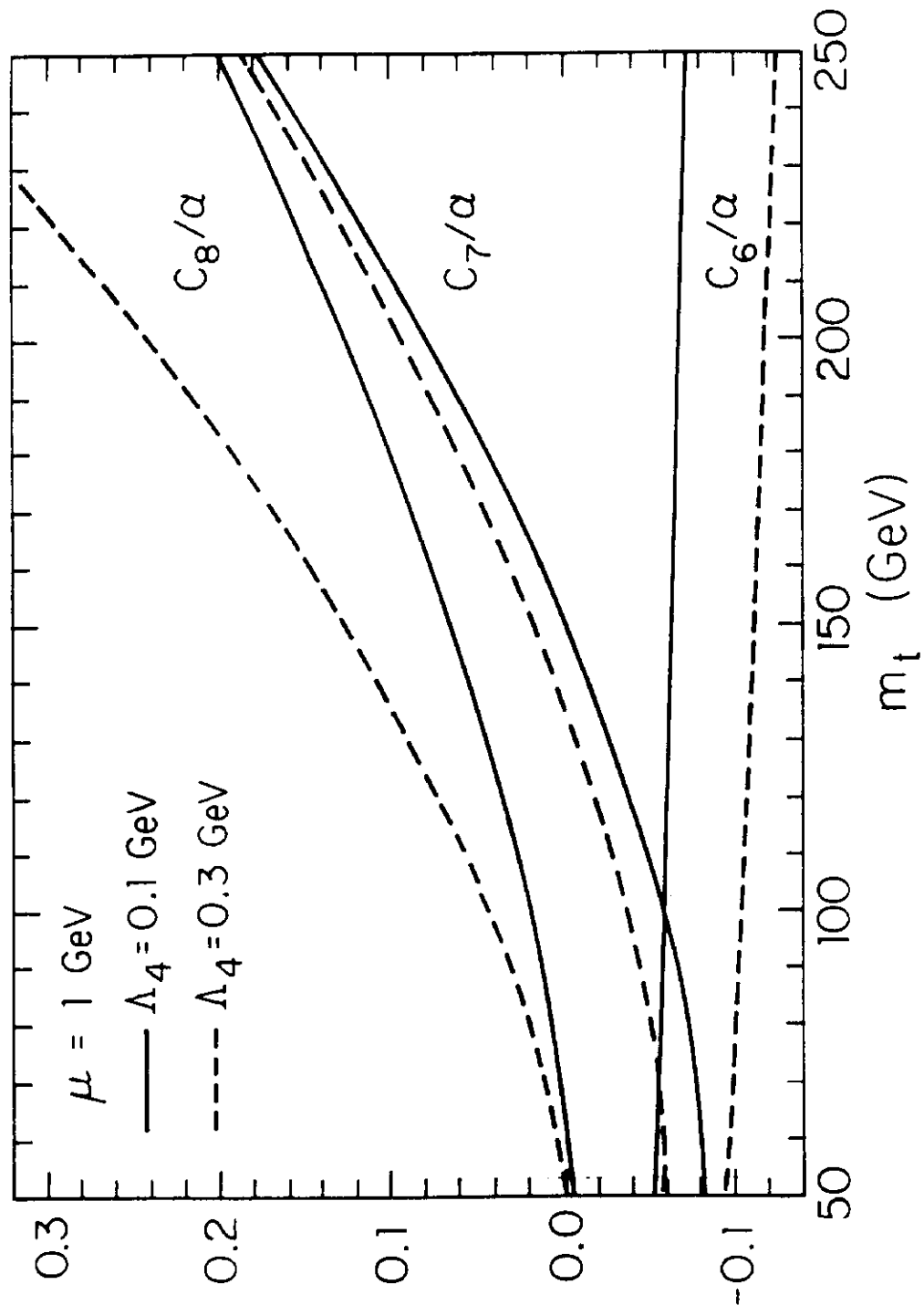


Fig. 2

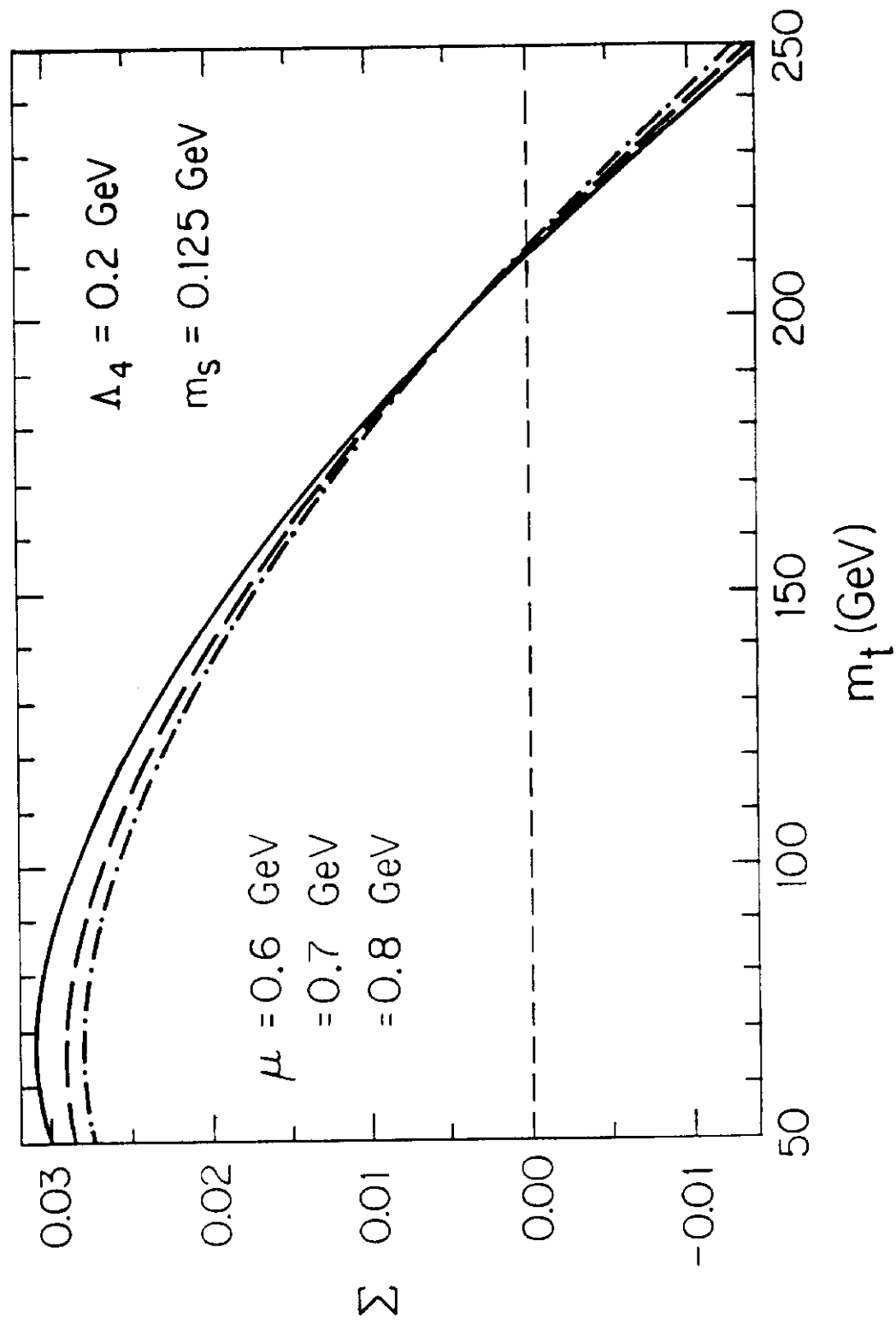


Fig. 3

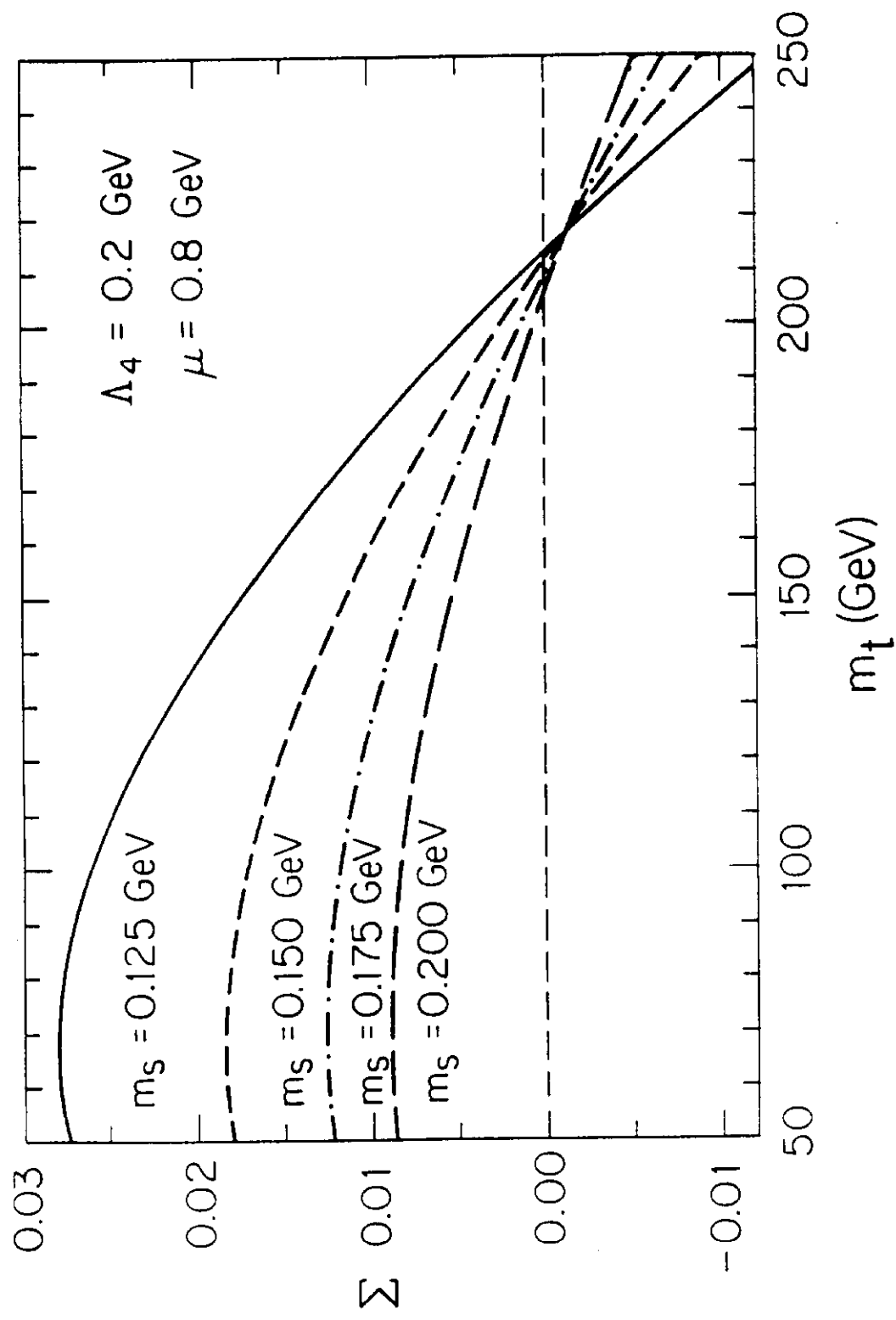


Fig. 4

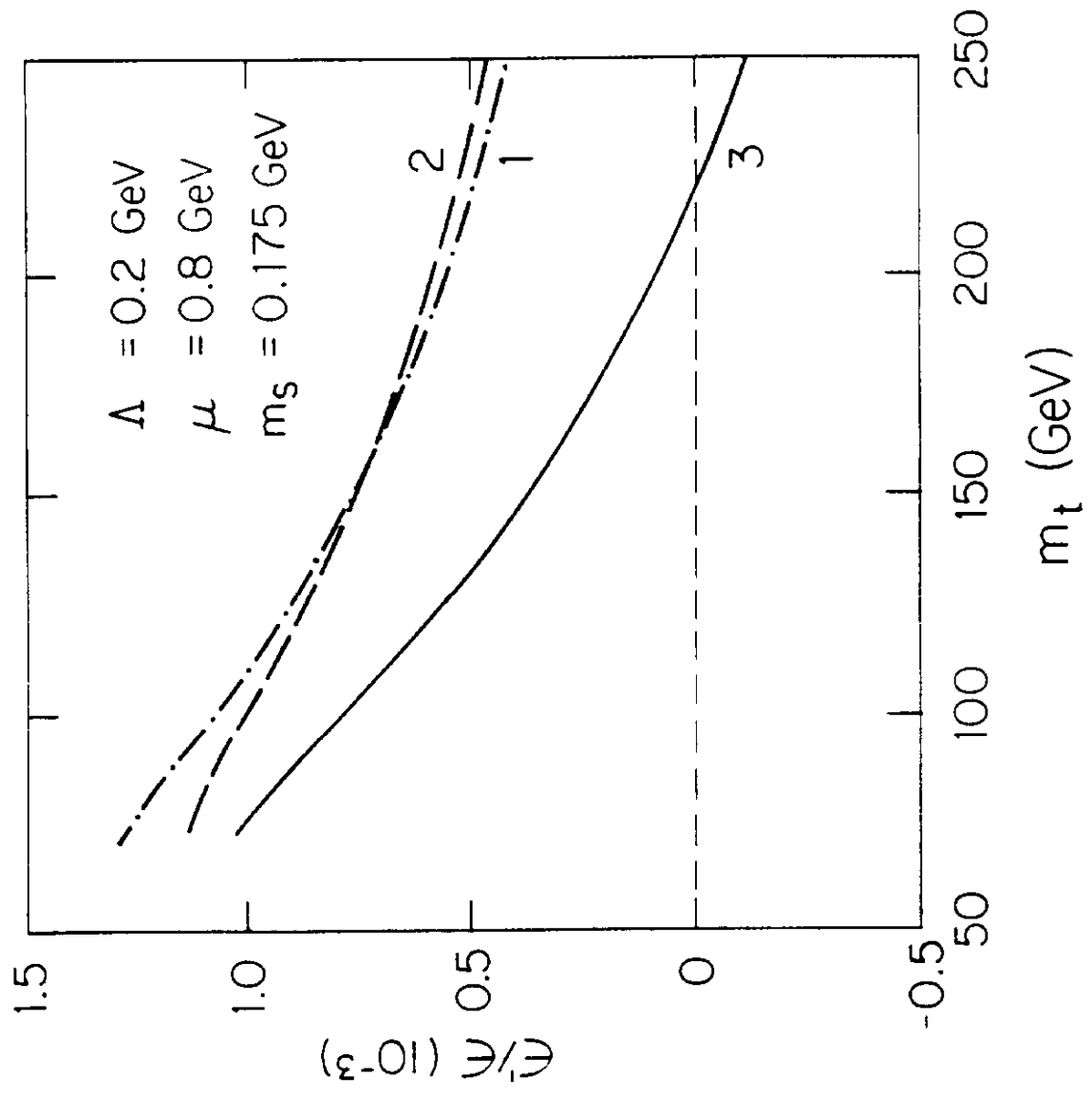


Fig. 5

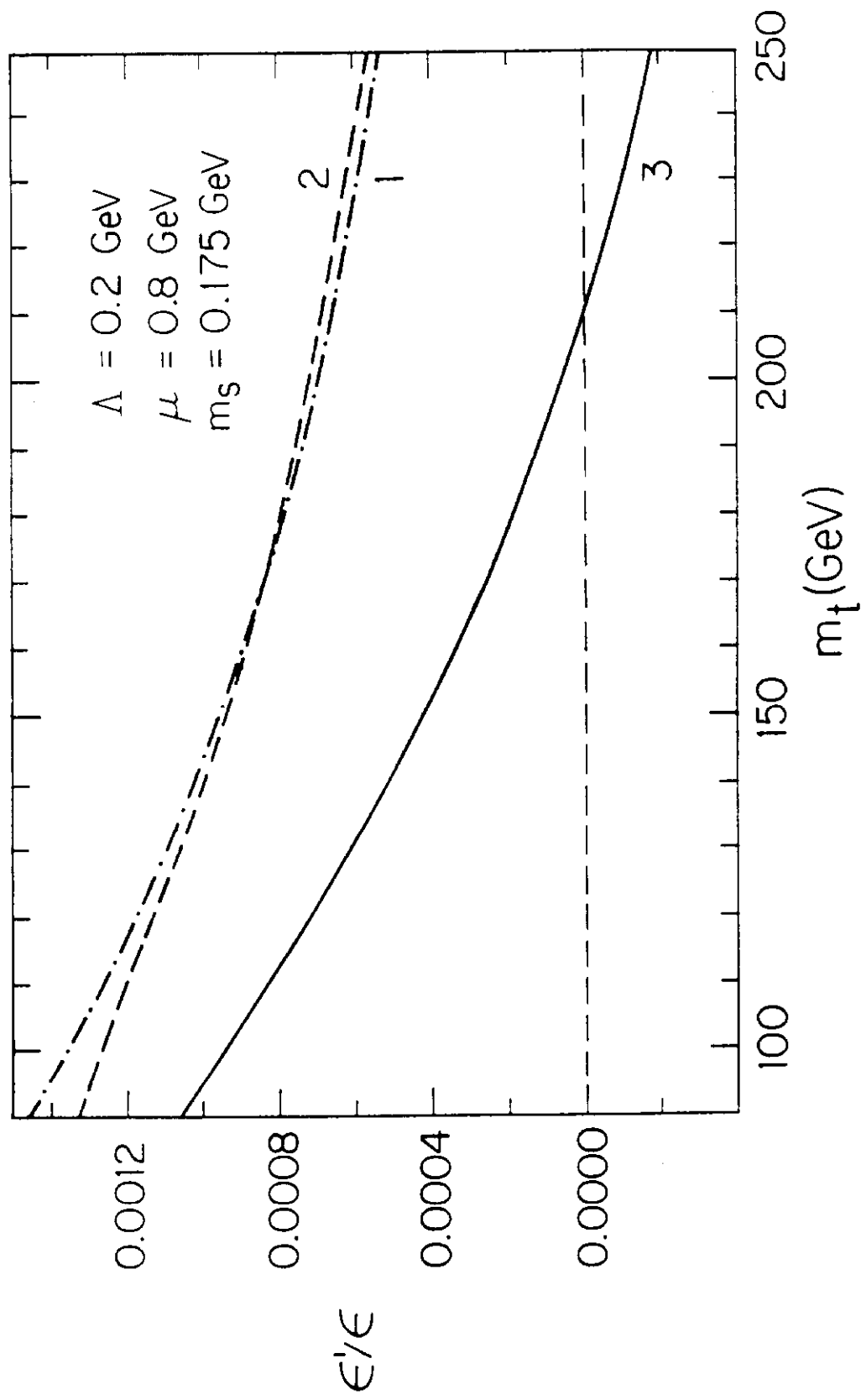


Fig. 6

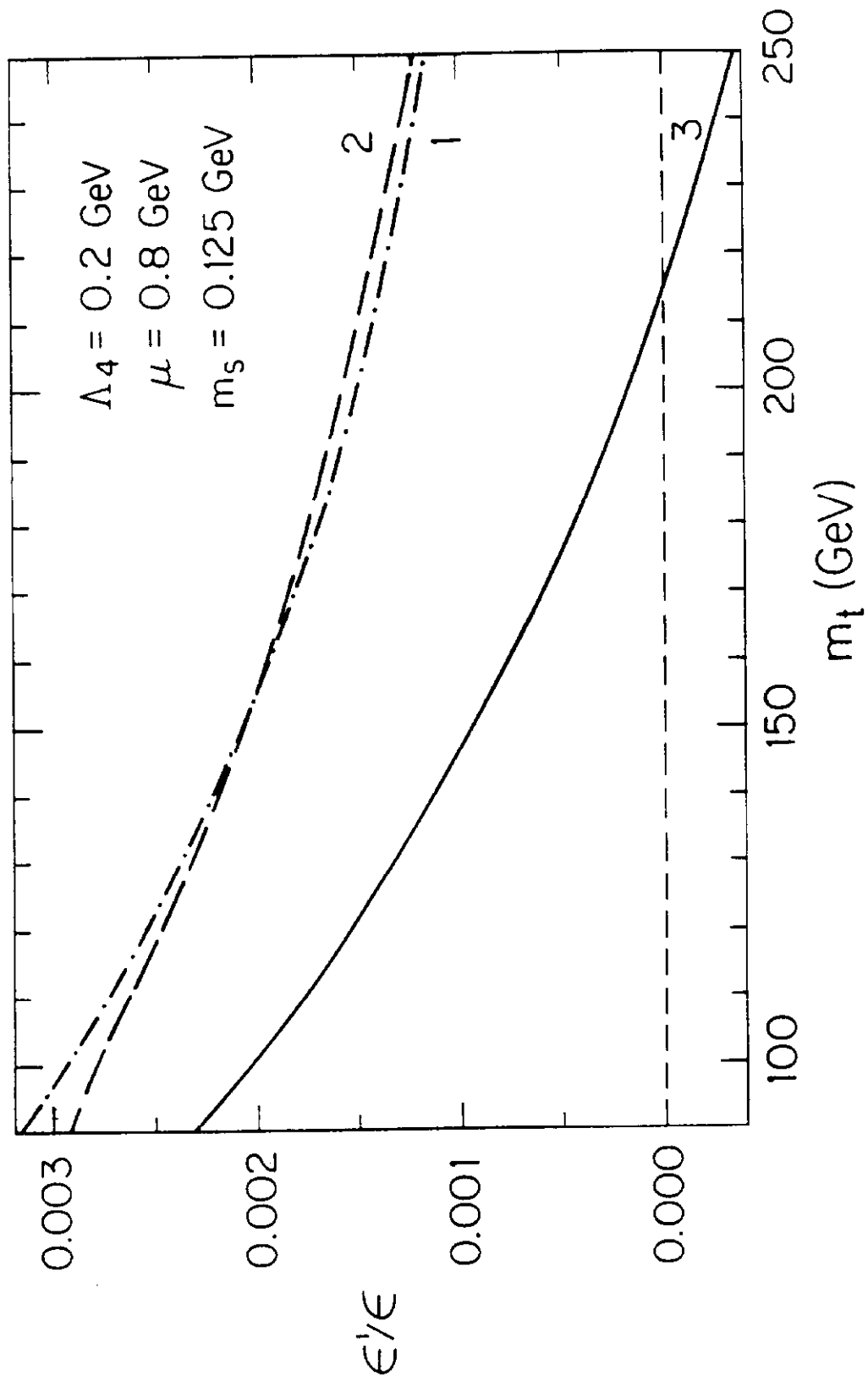


Fig. 7

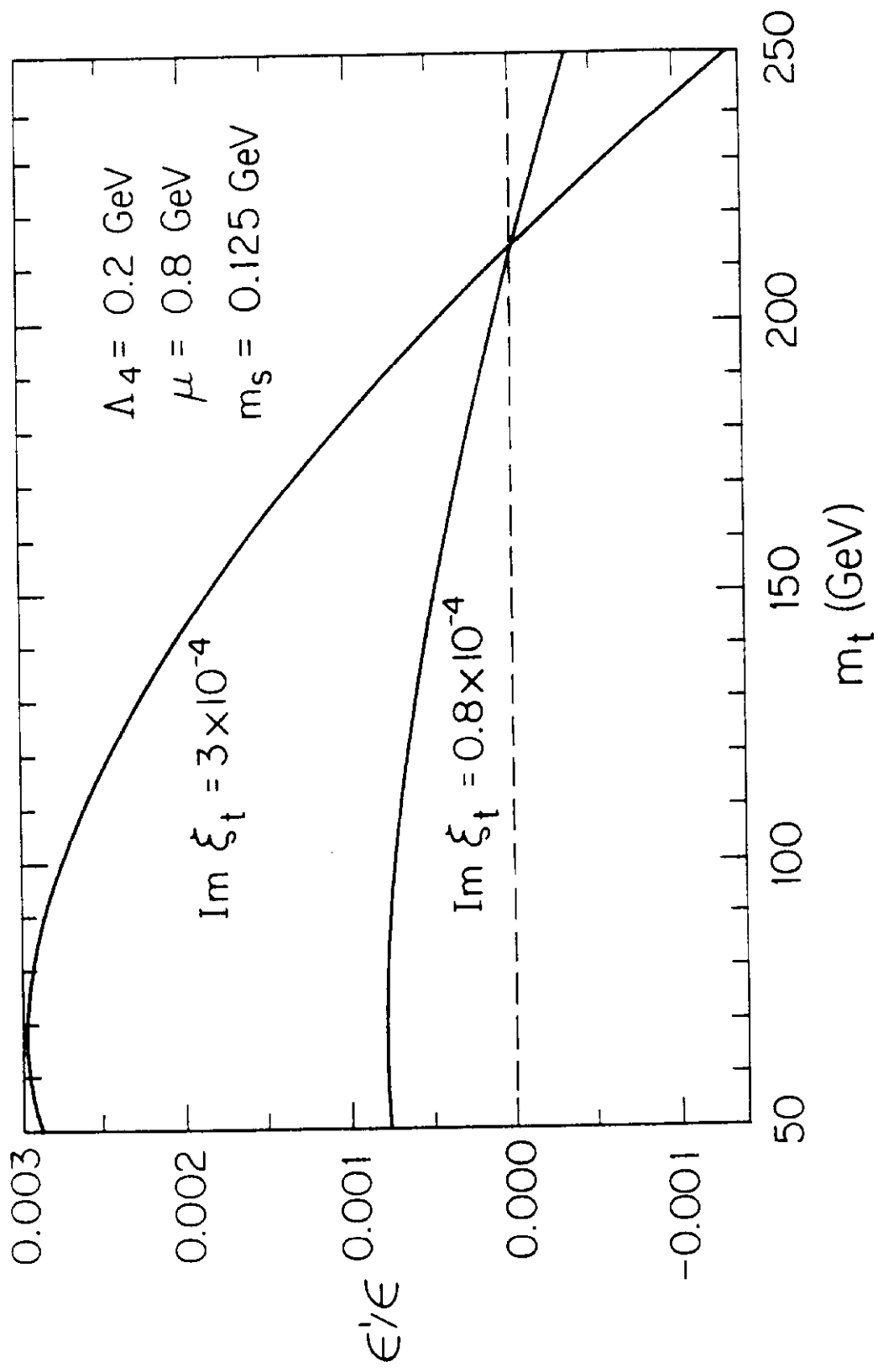


Fig. 8